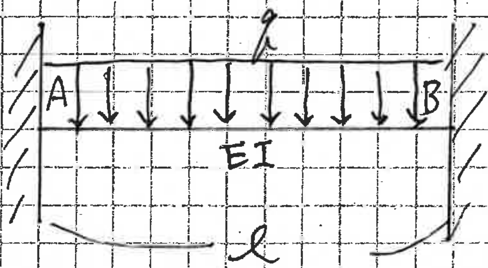
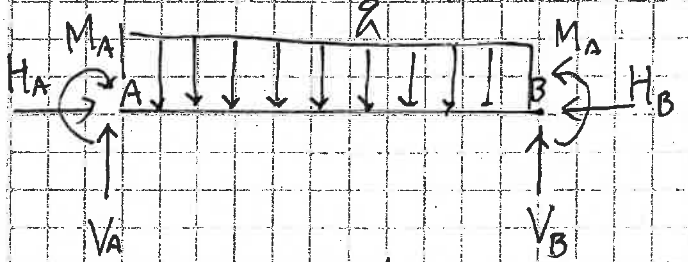
Q) 曲げモーメント図を作成せよ。 1/2



たわみ角法で解く。

◎材端モーメント式

$$\begin{cases} M_{AB} = \dfrac{2EI}{l}(2\theta_A+\theta_B-3R)+C_{AB} \quad \text{—①} \\ M_{BA} = \dfrac{2EI}{l}(\theta_A+2\theta_B-3R)+C_{BA} \quad \text{—②} \end{cases}$$



free body diagram

($\because H_A=H_B=0$)

◎境界条件

・A点とB点は固定である。

よって たわみ角 $\theta_A=\theta_B=0$。 —③

・A点とB点は不動点である

よって $R=0$ (部材角ゼロ) —④

◎ ③,④を①,②へ代入

$M_{AB}=C_{AB}$, $M_{BA}=C_{BA}$

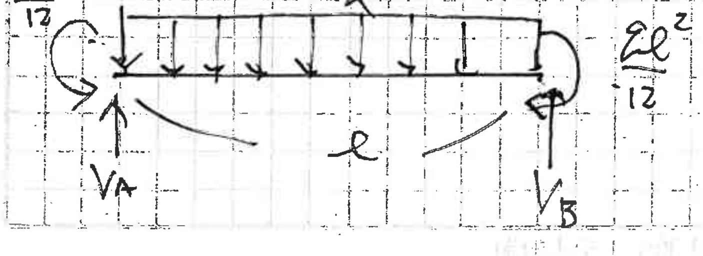
◎中間荷重項 C_{AB}, C_{BA} (表8.1 テキストP.157)

$$C_{AB} = -\frac{wl^2}{12}, \quad C_{BA} = \frac{wl^2}{12}$$

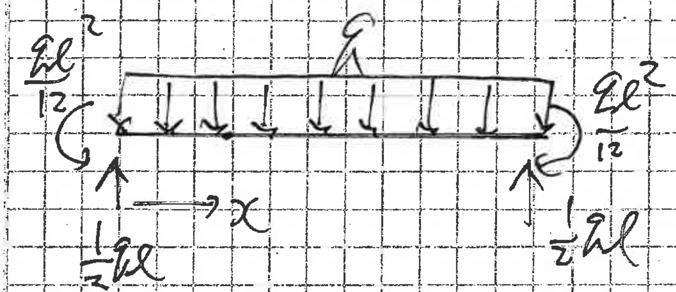
よって $M_{AB} = -\dfrac{wl^2}{12}$ // , $M_{BA} = \dfrac{wl^2}{12}$ //

梁理論における M_A と M_B を求める

$$M_A = M_{AB} = -\frac{wl^2}{12}, \quad M_B = -M_{BA} = -\frac{wl^2}{12}$$



$$\left(\begin{aligned} &\Sigma V = V_A+V_B-wl=0 \\ &\Sigma M_A = -V_B\times l-\frac{wl^2}{12}+\frac{wl^2}{12}+\frac{wl^2}{2}=0 \\ &V_B = \frac{1}{2}wl = V_B \end{aligned}\right.$$



$$M_x = -\frac{ql^2}{12} + \frac{1}{2}qlx - \frac{1}{2}qx^2$$

・$x=0$のとき

$$M_x = -\frac{ql^2}{12}$$

・$x=\frac{l}{2}$のとき

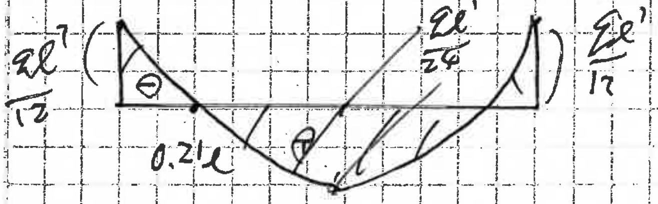
$$M_x = -\frac{ql^2}{12} + \frac{ql^2}{4} - \frac{ql^2}{8}$$

$$= \frac{-2+6-3}{24}ql^2$$

$$= \frac{1}{24}ql^2$$

・$x=l$のとき

$$M_x = -\frac{ql^2}{12}$$



〈曲げモーメント図〉

$$M_x = -\frac{ql^2}{12} + \frac{1}{2}qlx - \frac{1}{2}qx^2 = 0$$

$$-ql^2 + 6qlx - 6qx^2 = 0$$

$$6qx^2 - 6qlx + ql^2 = 0$$

解の公式 $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$a = 6q,\ b = -6ql,\ c = ql^2$

$$x = \frac{6ql \pm \sqrt{36q^2l^2 - 24q^2l^2}}{12q} = \frac{6ql \pm \sqrt{12q^2l^2}}{12q}$$

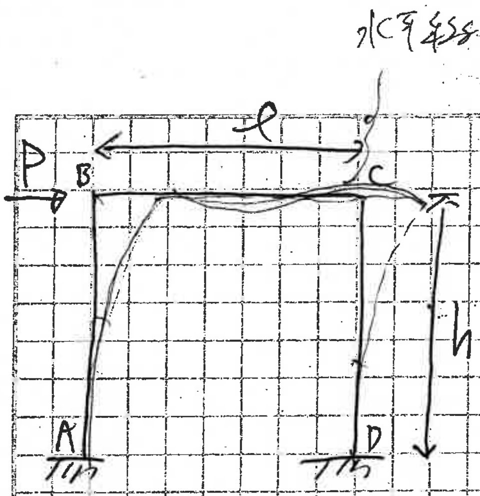
$$= \frac{6ql \pm 2ql\sqrt{3}}{12q} = \frac{3l \pm \sqrt{3}l}{6}$$

$$x = \frac{3l + \sqrt{3}l}{6} = \frac{3+\sqrt{3}}{6}l \fallingdotseq 0.7887l$$

$$= \frac{3l - \sqrt{3}l}{6} = \frac{3-\sqrt{3}}{6}l \fallingdotseq 0.2113l$$

問) 曲げモーメントの概略図を求めよ

図中 K=1 とする



◎ 境界条件

$\varphi_a = \varphi_d = 0,\ \psi_{bc} = \psi_{cb} = 0$

$C = 0$: 中間荷重なし

$\psi_{ab} = \psi_{dc} = \psi$ ($= \psi_{ba}$) ($= \psi_{cd}$)

◎ 材端モーメント式

$M_{ab} = 2\varphi_a + \varphi_b + \psi_{ab} + C_{ab}$

$= \varphi_b + \psi$ —①

$M_{ba} = 2\varphi_b + \varphi_a + \psi_{ba} + C_{ba}$

$= 2\varphi_b + \psi$ —②

同様に

$M_{bc} = 2\varphi_b + \varphi_c$ —③

$M_{cb} = 2\varphi_c + \varphi_b$ —④

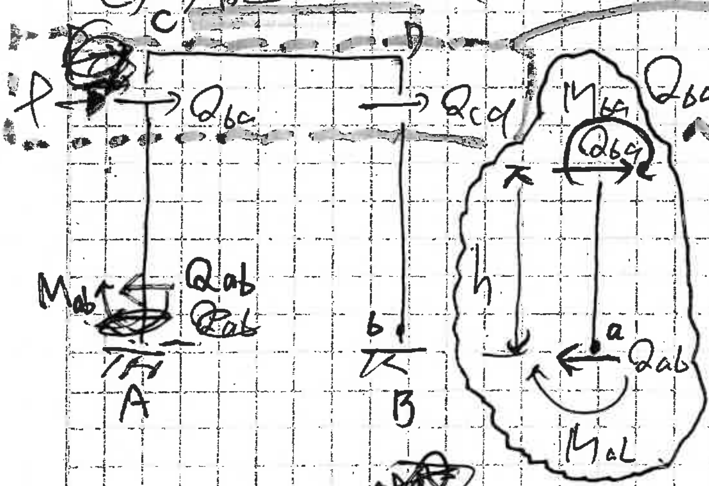
$M_{cd} = 2\varphi_c + \psi$ —⑤

$M_{dc} = \varphi_c + \psi$ —⑥

◎ 節点方程式 (変形適合条件)

$M_{ba} + M_{bc} = 0$ —⑦, $M_{cb} + M_{cd} = 0$ —⑧

◎ 層方程式 (せん断力のつり合い式)



$Q_{ba} + Q_{cd} = P$ —⑨

$\Sigma M_a = Q_{ba} h + M_{ba} + M_{ab} = 0$

$\therefore Q_{ba} = -\dfrac{M_{ba} + M_{ab}}{h}$ —⑩

$\Sigma M_b = Q_{cd} h + M_{cd} + M_{dc} = 0$

$\therefore Q_{cd} = -\dfrac{M_{cd} + M_{dc}}{h}$ —⑪

⑨に⑩,⑪を代入して

$\therefore -\dfrac{M_{ba} + M_{ab}}{h} - \dfrac{M_{cd} + M_{dc}}{h} = P$

$\therefore M_{ba} + M_{ab} + M_{cd} + M_{dc} = -Ph$

[illegible] ⑰ ⑱ に [illegible] ②, ③, ④, ⑤ を代入して

$$2\varphi_b + \psi + 2\varphi_b + \cancel{\psi}\ \varphi_c = 0 \quad — ⑬$$

$$\boxed{4\varphi_b + \varphi_c + \psi = 0}$$

$$2\varphi_c + \varphi_b + 2\varphi_c + \psi = 0$$

$$\boxed{\varphi_b + 4\varphi_c + \psi = 0} \quad — ⑭$$

$$2\varphi_b + \psi + \varphi_b + \psi + 2\varphi_c + \psi + \varphi_c + \psi = -ph$$

$$\boxed{3\varphi_b + 3\varphi_c + 4\psi = -ph} \quad — ⑮$$

⑬より $\varphi_c = -4\varphi_b - \psi$ — ⑯

⑯を⑭, ⑮へ代入

$$\varphi_b - 16\varphi_b - 4\psi + \psi = 0$$

$$15\varphi_b + 3\psi = 0$$

$$5\varphi_b + \psi = 0 \Rightarrow \underline{\psi = -5\varphi_b} \rightarrow ⑰$$

$$3\varphi_b - 12\varphi_b - 3\psi + 4\psi = -ph$$

$$-9\varphi_b + \psi = -ph \quad — ⑱$$

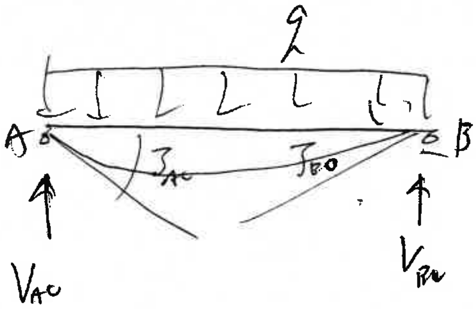
⑱に⑰を代入

$$-9\varphi_b - 5\varphi_b = -ph \qquad -14\varphi_b = -ph$$

$$\therefore \varphi_b = \frac{1}{14}ph \quad — ⑲$$

⑲を⑰へ代入して

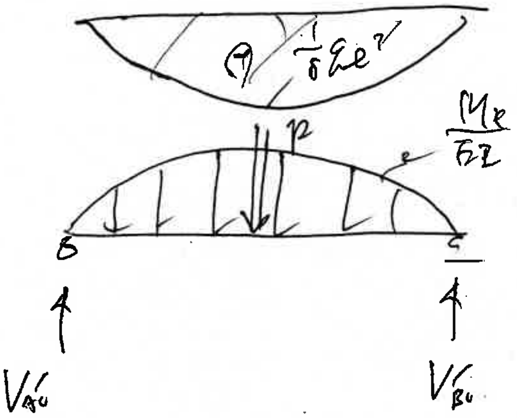
$$\psi = -5 \times \frac{1}{14}ph = -\frac{5}{14}ph \quad — ⑳$$



$$\Sigma M_A = -V_{BO}\,l + \frac{1}{2} q l^2 = 0$$

$$\therefore V_{BO} = \frac{1}{2} q l = V_{AO}$$

$$M_x = \frac{1}{2} q l x - \frac{1}{2} q x^2$$



$$P = \int_0^l \frac{M_x}{EI} dx$$

$$= \frac{1}{EI}\left[\frac{1}{4} q l x^2 - \frac{1}{6} q x^3\right]_0^l$$

$$= \frac{q l^3}{12EI}$$

$$\Sigma M_A = V'_{BO}\,l - \frac{q l^3}{12EI} \times \frac{l}{2} = 0$$

$$V'_{BO} = \frac{q l^3}{24EI} = V'_{AO} = \tau_{AO} = \tau_{BO}$$

$$\therefore C_{AB} = -\frac{2EI}{l}(2\tau_{AO} + \tau_{BO}) = -\frac{2EI}{l}\left(\frac{2ql^3}{24EI} - \frac{ql^3}{24EI}\right)$$

$$= -\frac{2}{l} \times \left(\frac{2-1}{24}\right) q l^3 = -\frac{q l^2}{12}$$

$$C_{BA} = -\frac{2EI}{l}(\tau_{AO} + 2\tau_{BO}) = -\frac{2EI}{l}\left(\frac{ql^3}{24EI} - \frac{2ql^3}{24EI}\right)$$

$$= \frac{1}{12} q l^2$$

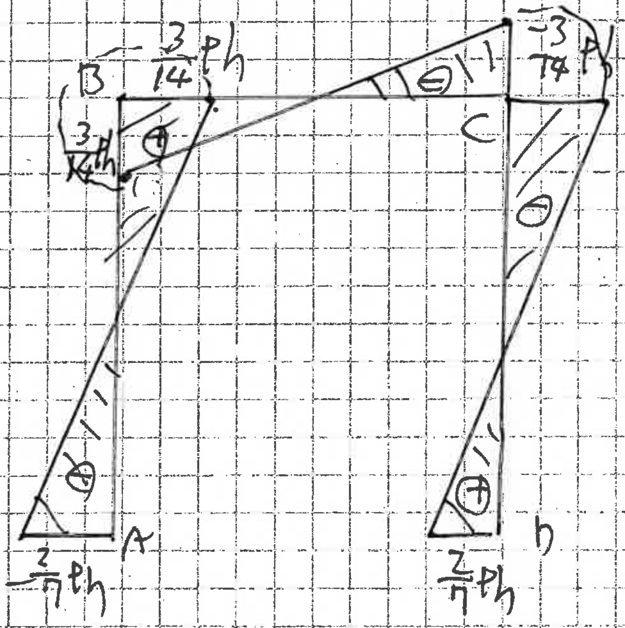
$$M_A = M_{AB} = \frac{2EI}{l}(\theta_A + 2\theta_B - 3R) + C_{AB}$$

境界条件より $\theta_A = \theta_B = R = 0,\ C_{AB} = -\frac{1}{12} q l^2$ より

$$M_A = -\frac{1}{12} q l^2$$

$$\begin{cases} \varphi_b = \dfrac{1}{14}Ph, \quad \varphi_c = -4 \times \dfrac{1}{14}Ph + \dfrac{5}{14}Ph = \dfrac{1}{14}Ph \\ \psi = -\dfrac{5}{14}Ph \end{cases}$$

$$\begin{cases} M_A = M_{ab} = \dfrac{1}{14}Ph - \dfrac{5}{14}Ph = -\dfrac{4}{14}Ph = -\dfrac{2}{7}Ph \\ M_B = M_{bc} = 2 \times \dfrac{1}{14}Ph + \dfrac{1}{14}Ph = \dfrac{3}{14}Ph \\ M_C = M_{cd} = 2 \times \dfrac{1}{14}Ph - \dfrac{5}{14}Ph = -\dfrac{3}{14}Ph \\ M_D = -M_{dc} = -\dfrac{1}{14}Ph + \dfrac{5}{14}Ph = \dfrac{4}{14}Ph = \dfrac{2}{7}Ph \end{cases}$$



〈曲げモーメント図〉