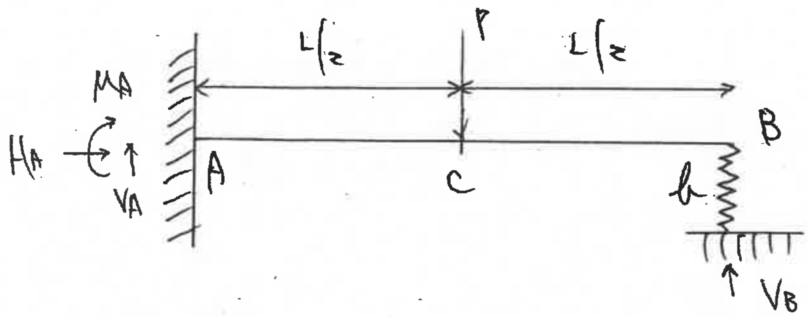
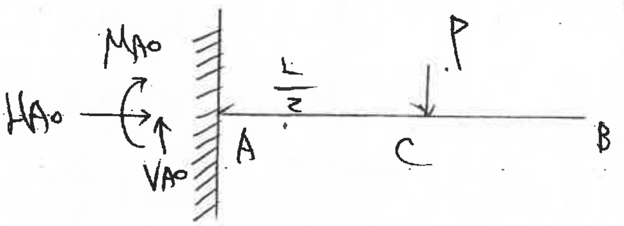
問題6　B点がバネ支持　A点が固定支持　C点に荷重P　EIは一定

点Bのたわみ



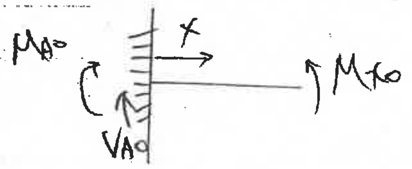
(0系)　B点の拘束解除



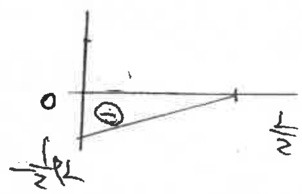
・支点反力

$$\begin{cases} \Sigma H : H_{A0} = 0 \\ \Sigma V : V_{A0} - P = 0 \qquad V_{A0} = P \\ \Sigma M_{(A)} : M_{A0} + \frac{PL}{2} = 0 \qquad M_{A0} = -\frac{PL}{2} \end{cases}$$

(A < x < C)

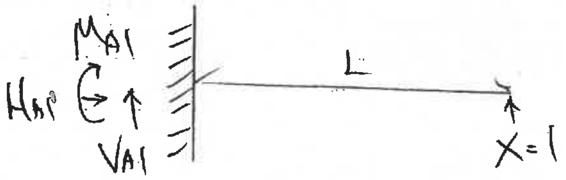


<M図>



$$\Sigma M : V_{A0}x + M_{A0} - M_{x0} = 0$$
$$M_{x0} = Px - \frac{1}{2}PL$$

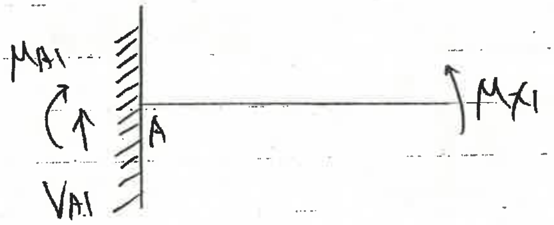
(1系)　B点に X=1 を作用



・支点反力

$$\begin{cases} \Sigma H : H_{A1} = 0 \\ \Sigma V : V_{A1} + 1 = 0 \qquad V_{A1} = -1 \\ \Sigma M : M_{A1} - L = 0 \qquad M_{A1} = L \end{cases}$$

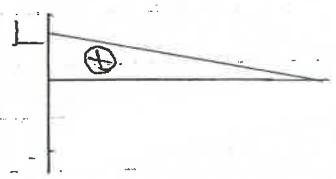
(A < x < B)



$\Sigma M: V_{A1} x - M_{x1} + M_{A1}$

$M_{x1} = -x + L$

<M図>



バネ系

フックの法則より

$F = kx$

$X = k\Delta_k$

$\Delta_k = \frac{X}{k}$

変形適合条件式より、

$$\Delta_{10} + \Delta_{11} X_1 + \frac{X_1}{k} = 0 \quad \cdots ① \quad (\Delta_B = \Delta_{10} + \Delta_{11} X_1 + \Delta_k = 0)$$

$$\Delta_{10} = \frac{1}{EI}\int_0^{\frac{L}{2}} M_{x1} M_{x0}\, dx = \frac{1}{EI}\int_0^{\frac{L}{2}} \left(Px - \frac{PL}{2}\right)(-x+L)\, dx = -\frac{5PL^3}{48EI}$$

$$\Delta_{11} = \frac{1}{EI}\int_0^{L} (-x+L)^2\, dx = \frac{L^3}{3EI}$$

① より、 $$X_1 = -\frac{\Delta_{10}}{\Delta_{11} + \frac{1}{k}} = \frac{\frac{5PL^3}{48EI}}{\frac{L^3}{3EI} + \frac{1}{k}} = \frac{5PL^3 k}{16L^3 k + 48EI}$$

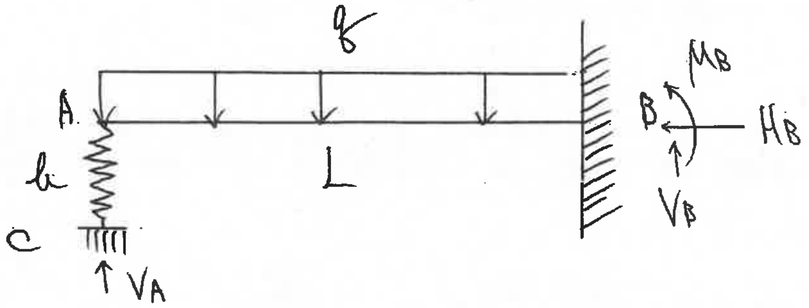
$\Delta_B = \frac{X_1}{k}$ より、

$$= \frac{1}{\cancel{k}} \times \frac{5PL^3 \cancel{k}}{16L^3 k + 48EI} = \frac{5PL^3}{16(L^3 k + 3EI)}$$

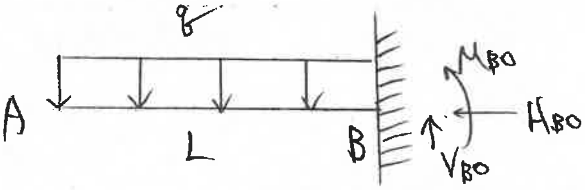
よって、B点のたわみ

$$\Delta_B = \frac{5PL^3}{16(L^3 k + 3EI)}$$

問題7　ばねに作用する力　曲げ剛性 EI 一定



(0系)　A点の拘束解除



○支点反力

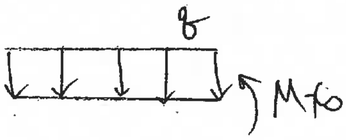
$\Sigma H = H_{B0} = 0$

$\Sigma V = V_{B0} - qL = 0 \qquad V_{B0} = qL$

$\underset{(B)}{\Sigma M} = -M_{B0} - \frac{L}{2} \cdot qL = 0$

$M_{B0} = -\frac{1}{2} qL^2$

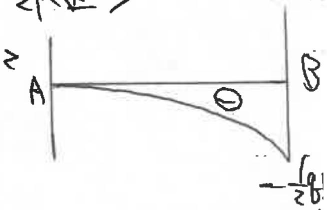
(A < x < B)



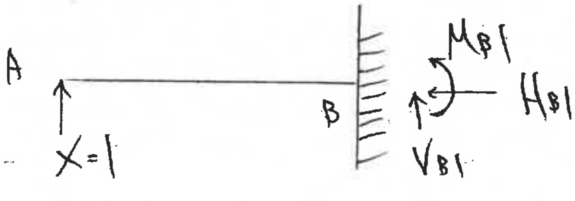
$\Sigma M = -M_{x0} - \frac{1}{2} x \cdot qx = 0$

$M_{x0} = -\frac{1}{2} qx^2$

<M図>



(1系)　A点に X = 1 を作用



○支点反力

$\Sigma H = H_{B1} = 0$

$\Sigma V = V_{B1} + 1 = 0 \qquad V_{B1} = -1$

$\underset{(B)}{\Sigma M} = L - M_{B1} = 0$

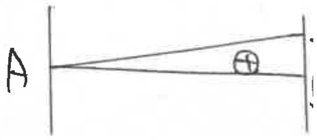
$M_{B1} = L$

(A < x < B)

$\Sigma M = -M_{x1} + x = 0$

$M_{x1} = x$

<M図>



変形適合条件式より、

$$\Delta_A = \delta_{10} + \delta_{11} X_1 + \frac{X_1}{k} = 0 \quad \sim ①$$

$$\delta_{10} = \frac{1}{EI}\int_0^L M_{x1} M_{x0}\, dx = \frac{1}{EI}\int_0^L \left(-\frac{1}{2}qx^2\right)(x)\, dx = -\frac{qL^4}{8EI}$$

$$\delta_{11} = \frac{1}{EI}\int_0^L M_{x1} M_{x1}\, dx = \frac{1}{EI}\int_0^L x^2\, dx = \frac{L^3}{3EI}$$

①より、

$$X_1 = -\frac{\delta_{10}}{\delta_{11} + \frac{1}{k}} = \frac{\frac{qL^4}{8EI}}{\frac{L^3}{3EI} + \frac{1}{k}} = \frac{3qL^4k}{8L^3k + 24EI}$$

よって、ばねに作用する力は

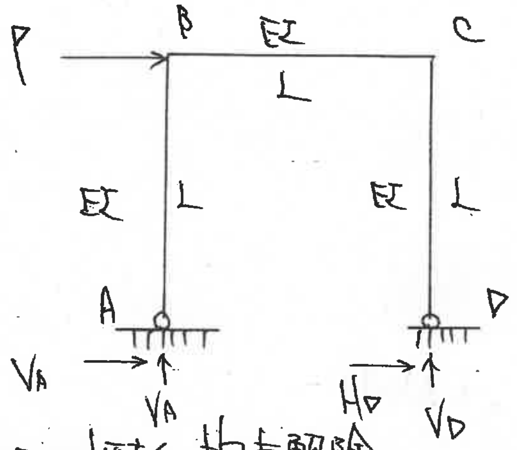
$$\frac{3qL^4k}{8(L^3k + 3EI)}$$

c)

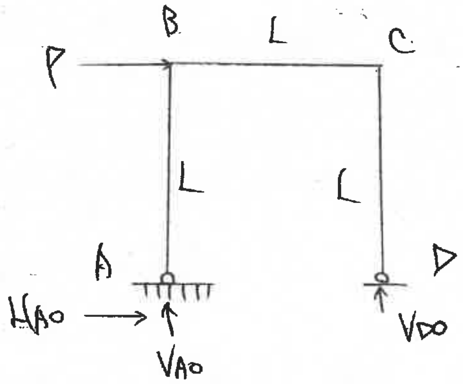
バネ系

$$\Delta_k = \frac{X}{k}$$

問題8　　全ての支点反力　　EI は一定



(0系) 点Dの水平方向拘束解除



・支点反力

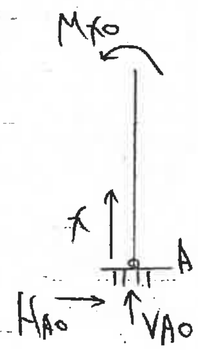
$\Sigma H : H_{A0} + P = 0 \qquad H_{A0} = -P$

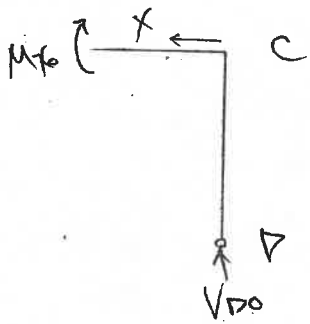
$\Sigma V : V_{A0} + V_{D0} = 0$

$\underset{(A)}{\Sigma M} : PL - V_{D0} L = 0$

$V_{D0} = P \ , \quad V_{A0} = -P$

$(A < x < B)$　　　　$(D < x < C)$



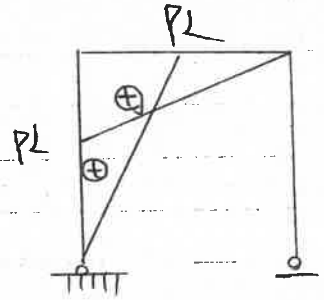


$\Sigma M : -M_{x0} - H_{A0} x = 0$
$M_{x0} = Px$

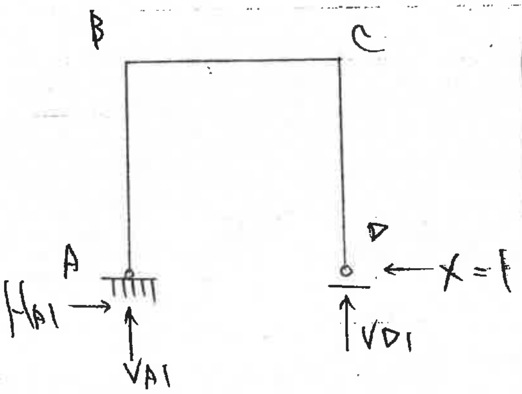
$\Sigma M : M_{x0} = 0$

$\Sigma M : M_{x0} - V_{D0} x = 0$
$M_x = Px$

<M図>



(1系) 点Dの水平方向に $X=1$ を作用



。支点反力

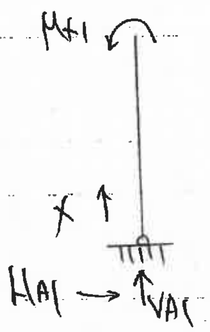
$\Sigma V = V_{A1} + V_{D1} = 0$　　$V_{A1} = 0$

$\Sigma H = H_{A1} - 1 = 0$　　$H_{A1} = 1$

$\Sigma M_{(A)} = -V_{D1}L = 0$

$V_{D1} = 0$

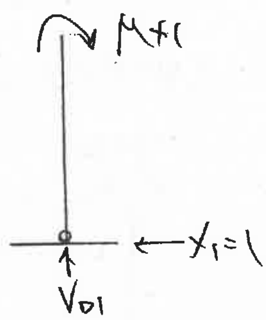
(A < x < B)



$\Sigma M = -M_{x1} - H_{A1}x = 0$

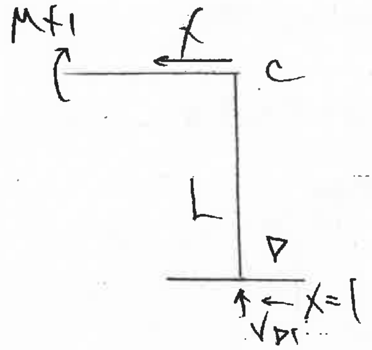
$M_{x1} = -x$

(D < x < C)



$\Sigma M = M_{x1} + 1 \cdot x = 0$

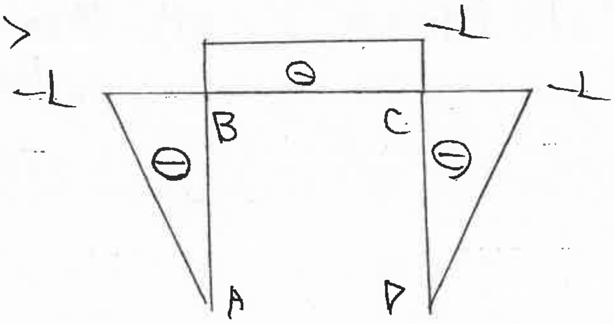
$M_{x1} = -x$



$\Sigma M = M_{x1} - V_{D1}L + 1 \cdot L = 0$

$M_{x1} = -L$

<M図>



得られた結果をそれぞれ記す

	L	M_0	M_1
A〜B	L	Px	$-x$
D〜C	L	0	$-x$
C〜B	L	Px	$-L$

変形適合条件式より、

$$\delta_{10} + \delta_{11} X_1 = 0$$

$$\delta_{10} = \frac{1}{EI}\left\{\int_0^L (Px)(-x)\,dx + \int_0^L (0)(-x)\,dx + \int_0^L (Px)(-L)\,dx\right\}$$

$$= \frac{1}{EI}\left\{\left[-\frac{1}{3}Px^3\right]_0^L + \left[-\frac{Px^2L}{2}\right]_0^L\right.$$

$$= -\frac{5PL^3}{6EI}$$

$$\delta_{11} = \frac{1}{EI}\left\{\int_0^L (x^2)\,dx + \int_0^L (x^2)\,dx + \int_0^L (L^2)\,dx\right\}$$

$$= \frac{1}{EI}\left\{\left[\frac{1}{3}x^3\right]_0^L + \left[\frac{1}{3}x^3\right]_0^L + \left[L^2x\right]_0^L\right.$$

$$= \frac{5L^3}{3EI}$$

① にそれぞれ代入して、

$$X_1 = -\frac{\delta_{10}}{\delta_{11}} = \frac{\cancel{5}P\cancel{L^3}}{6\cancel{EI}} \times \frac{3\cancel{EI}}{\cancel{5}\cancel{L^3}} = \frac{P}{2}$$

$X_1 = \frac{P}{2}$ より、

$$V_A = V_{A0} + V_{A1}X_1 = -P + 0 \times \frac{P}{2} = -P$$

$$H_A = H_{A0} + H_{A1}X_1 = -P + 1 \times \frac{P}{2} = -\frac{P}{2}$$

$$V_D = V_{D0} + V_{D1}X_1 = P + 0 \times \frac{P}{2} = P$$

$$H_D = H_{D0} + H_{D1}X_1 = 0 + \frac{P}{2} \times 1 = \frac{P}{2}$$

よって、支点反力は

$V_A = -P$
$H_A = -\frac{P}{2}$
$V_D = P$　　となる。
A、 $H_D = \frac{P}{2}$